Comparative Analysis of Local & Global Optimal Fuzzy Controller for Mass Spring Damper System

Sanjay Charaya, Lecturer
Department of Electrical Engineering, Ch. Devi Lal Memorial Government Engineering College, PanniwalaMota, Sirsa, Haryana (India)

ABSTRACT: In this paper, local concept approach and the global concept approach have been reviewed. Firstly, the procedure for designing the optimal controller via local concept approach is listed. The stability of the entire closed-loop continuous fuzzy system is ensured. Then a systematic way to design a global optimal fuzzy controller and stabilizing a continuous fuzzy system is presented by viewing in local and global concepts. A linear-like global system representation is considered by viewing the fuzzy system in global concept by unifying the individual matrices into synthetical matrices. A comparative analysis under the two approaches is made. Stability has been demonstrated for both approaches while considering a mass-spring-damper mechanical system.

KEYWORDS–Global optimal, Riccati equation, Riccati-like equation, T-S type fuzzy model

I. INTRODUCTION

Conventional control algorithms require a mathematical model of the dynamical system to be controlled. The mathematical model is then used to construct a controller. In many practical situations, however, it is not always feasible to obtain an accurate mathematical model of the controlled system. Fuzzy logic control offers a way of dealing with modeling problems by implementing linguistics which are nonformly expressed control laws that are derived from expert knowledge. The fuzzy control is more effective in dealing with real systems than the linear control theory. Moreover, the optimal control provides a most effective control strategy. Therefore, it is quite interesting to investigate the optimal fuzzy controller design concepts.

One of the design techniques of Takagi-Sugeno (T-S) type fuzzy modeling and control is based on parallel distributed compensation (PDC) of nonlinear system [1]-[3] s. In this concept each control rule is distributively designed for the corresponding rule of a T-S fuzzy model. For each rule, linear control design techniques can be used. The resulting overall controller is a fuzzy blending of each individual linear controller. The fuzzy controller, so obtained, is nonlinear in general and applies Lyapunov's method to do stability analysis. [1] The stability analysis and control design problems are reduced to linear matrix inequality problems. A fuzzy controller is designed based on the LMI stability conditions. This approach had been applied to several control problems in [4], [5], [6]. In [7] the stability conditions are relaxed with constraints on control input and output in the optimal design. [8] A T-S fuzzy model based fuzzy controller and fuzzy observer are designed. [9] The Linear matrix inequality (LMI) based design of fuzzy regulator and fuzzy observer are presented.

In the field of optimal control is, a fuzzy optimal controller is designed by Wang, to stabilize a linear time invariant system [10]. The Pontryagin maximum principle is used in design procedure. This design technique has the limitation that it is not good for nonlinear systems and therefore not has much practical implications. Tanaka, Taniguchi and Wang shown an LMI based procedure to optimal fuzzy control by solving the minimization problem that minimizes the upper bound of a given quadratic performance function [11]. Wu and Lin proposed two approaches named local concept approach and global concept approach of optimal controller design [12] – [14]. In local concept approach, an optimal fuzzy controller design is achieved from a local view point. The controller, so designed, exposes properties based on the linear optimal control theory. Based on this concept, an optimal fuzzy tracker has been designed [15]. The global concept approach, applies to continuous time systems [16]. It presents a different procedure for controller design. It proposes a linear-like system representation for the fuzzy system problem via unifying the individual matrices into synthetical matrices. The derived control law is demonstrated to be the best for the entire system to reach the optimal performance index. The approach is then applied to the discrete time systems [17].

Genetic algorithm (GA) was first introduced by J. Holland as search algorithm [18]. Its extension is used in fuzzy optimal controller design. The use of GA in the design of a fuzzy controller not only provides the global benefits of GA's, but also develops a systematic design approach for the fuzzy controller. Ho et.al. Integrated the orthogonal function approach (OFA) and the GA to study the quadratic optimal design problems of both the fuzzy PDC controller and the non-PDC controller (linear-state feedback controller) for the TS fuzzy model-based control systems [19]. This proposed technique involves the time-consuming inversion of large dimension matrices as a result of the Kronecker matrix product during solving the feedback dynamic equations. They further proposed a new method, which integrated the OFA and the hybrid Taguchi-genetic algorithm, to design the quadratic optimal controller [20]. This method did not use the Kronecker matrix product. An optimal controller was designed by the dynamic programming approach and the inverse optimal approach subject to the constraint on control inputs for continuous time T-S fuzzy systems [21].

This paper comprises of application of local concept approach and global concept approach to the optimal fuzzy controller design. A common example is also given to illustrate the both approaches and demonstrates the stability. In section III the comparison of the two approaches is described. The two design methodologies are applied to example of mass spring damper mechanical system in section IV. Section V gives the concluding remarks.

II. BRIEF REVIEW

Consider a non-linear plant described by the so called T-S type fuzzy model;

$$R^i: \text{If } x_1 \text{ is } T_{1i},\ldots\ldots., x_n \text{ is } T_{ni}\text{, Then } \dot{X}(t) = A_i(t)X(t) + B_i(t)u(t);$$
$$Y(t) = C(t)X(t), i = 1, 2, \ldots\ldots\ldots, r \tag{1}$$

where, $R^i \rightarrow$ i^th rule of the fuzzy model.

$x_1,\ldots\ldots., x_n \rightarrow$ system states

$T_{1i},\ldots\ldots., T_{ni} \rightarrow$ input fuzzy terms in the i^th rule.

$X(t) = \left[x_1,\ldots., x_n\right]^T \rightarrow$ state vector

$Y(t) \rightarrow$ system output vector

$A_i(t), B_i(t)$ and $C(t)$ respectively, nxn, nxm and n'xn matrices whose elements are known to be piecewise continuous (PC) and real-valued functions defined on positive real space. The desired controller is a rule-based nonlinear fuzzy controller given as;

$$R^i: \text{If } y_1 \text{ is } S_{1i},\ldots\ldots., y_n \text{ is } S_{ni} \text{ Then } u(t) = r_i(t),\ i = 1,\ldots., \delta \tag{2}$$

where, $y_1\ldots., y_n \rightarrow$ elements of output vector ;

$S_{1i},\ldots., S_{ni} \rightarrow$ input fuzzy terms in the i^th control rule;

$u(t)$ or $r_i(t) \rightarrow$ plant input (i.e., control output) vector.

The problem is to, find a controller $u^*(.)$, which can minimize the quadratic cost functional;

$$J(u(.)) = \int_{t0}^{t1} \left[X^T(t)L(t)X(t) + u^T(t)u(t)\right]dt + X^T(t_1)QX(t_1); t \in \left[t_0 \quad t_1\right] \tag{3}$$

over all possible inputs $u(.)$ of class piecewise-continuous.

Local concept approach:

Finite-Horizon Problem: For the fuzzy system in (1) and fuzzy controller in (2), let

$$A_i(t), B_i(t), C(t), L(t) = L^T(t) \geq 0, Q = Q^T \geq 0$$

be given matrices. If there exists on an symmetric positive semidefinite solution to the matrix Riccati differential equation

$$\dot{K}(t) = -A_i^T(t)K(t) - K(t)A_i(t) + K(t)B_i(t)B_i^T(t)K(t) - L(t) \tag{4}$$

where the final value of the dependent variable $K(t), K(t_1)$ is equal to the final state penalty index Q, and , then there exists a local optimal fuzzy control law

$$r_i^* = -B_i^T\pi^i(t, Q, t_1)X^*(t), i = 1,\ldots, r \tag{5}$$

where $X^*(t)$ is the corresponding optimal state trajectory. And, the corresponding global minimizer is

$$u^*(t) = \sum_{i=1}^{r} h_i(X^*(t)r_i^*(t) \tag{6}$$

which minimizes J(u(.)) in (3). The resulting optimal closed-loop system dynamics is described by

$$\dot{X}^*(t) = \sum_{i=1}^{r} h_i(X^*(t)) \left[A_i(t) - B_i(t) B_i^T(t) \pi^i(t,Q,t_1) \right] X^*(t); t \in \left[t_0 \quad t_1 \right] \tag{7}$$

with $X(t_0) = X_0$. The above theorem considers that the horizon t_1 is fixed and $t_0 \in \left[0, t_1\right)$ is arbitrary.

Infinite-Horizon problem: For the fuzzy system in (1) and fuzzy controller in (2), let A_i,B_i,C,L be given constant matrices and L = C^TC . If (A_i,B_i) is completely controllable (c.c.) and (A_i,C) is completely observable (c.o.) for i=1,………,r, then

1) There exists a unique n x n symmetric positive semi definite solution, π_∞^i, of the steady-state Riccati equation (S.S.R.E.)

$$A_i^T K + K A_i - K B_i B_i^T K + C^T C = 0 \tag{8}$$

2) The asymptotically local optimal fuzzy control law is

$$r_i^*(t) = -B_i^T \pi_\infty^i X^*(t), i = 1, ..., r. \tag{9}$$

and their "blending" global minimizer

$$u^*(t) = \sum_{i=1}^{r} h_i(X^*(t)) r_i^*(t) \tag{10}$$

minimizes J(u(.)) in equation (3).

3) and the optimal local feedback fuzzy subsystem

$$X^*(t) = (A_i - B_i B_i^T \pi_\infty^i) X^*(t) \tag{11}$$

is asymptotically and exponentially stable.

Global concept approach:

This approach formulates the distributed fuzzy subsystems and rule based fuzzy controller into one equation given as;

$$\dot{X}(t) = \sum_{i=1}^{r} h_i(X(t)) A_i(t) X(t) + \sum_{i=1}^{r} h_i(X(t)) B_i(t) u(t)$$

$$Y(t) = \sum_{i=1}^{r} h_i(X(t)) C(t) X(t)$$

and the entire fuzzy controller is

$$u(t) = \sum_{i=1}^{\delta} w_i(Y(t)) r_i(t) \text{ with } \sum_{i=1}^{r} h_i(X(t)) = 1 \text{ and } \sum_{i=1}^{\delta} w_i(Y(t)) = 1$$

where $h_i(X(t))$ and $w_i(Y(t))$ denote, respectively, the normalized firing-strength of the i^{th} rule of the continuous fuzzy model and that of the i^{th} fuzzy control rule. Introducing the following synthetical matrices, H(X(t)), W(Y(t)), A(t), B(t) and R(t). where

$$H\left(X\left(t\right)\right) = \left[\; h_1\left(X\left(t\right)\right) I_n ....h_r\left(X\left(t\right)\right) I_n \;\right] \tag{12}$$

$$W\left(Y\left(t\right)\right) = \left[\; w_1\left(Y\left(t\right)\right) I_m ....w_r\left(Y\left(t\right)\right) I_m \;\right] \tag{13}$$

$$A(t) = \begin{bmatrix} A_1(t) \\ . \\ . \\ . \\ A_r(t) \end{bmatrix}, \quad B(t) = \begin{bmatrix} B_1(t) \\ . \\ . \\ . \\ B_r(t) \end{bmatrix}, \quad R(t) = \begin{bmatrix} r_1(t) \\ . \\ . \\ . \\ r_\delta(t) \end{bmatrix} \tag{14}$$

with I_n and I_m denoting the identity matrices of dimensions n and m respectively. Based on these synthetical notations, the original problem can be rewritten as a nonlinear but linear-like closed-loop fuzzy system;

$$\begin{aligned} \dot{X}(t) &= H(X(t))A(t)X(t) + H(X(t))B(t)W(Y(t))R(t) \\ Y(t) &= C(t)X(t) \end{aligned} \tag{15}$$

with $X(t_0) = X_0$, finding the optimal synthetical control law, $R^*(.)$ to minimize the quadratic cost functional

$$J^i(R(.)) = \int_{t_0^i}^{t_1^i} \left(X^t(t)L(t)X(t) + R^t(t)W^t(Y(t))W(Y(t))R(t) \right) dt + X^t(t_1^i)Q^i X(t_1^i) \tag{16}$$

This linear-like synthetical matrix representation for the entire T–S type fuzzy system materializes the design of the global optimal fuzzy controller in the way of general LQ approach, i.e., calculus-of-variation method.

Finite-Horizon Problem: Consider the time-invariant fuzzy system and fuzzy controller described, respectively, by (1) and (2) with $L = C^T C$ in (16). Let $\left(X^*(t), R^*(t)\right), t \in [t_0, t_1]$, denote the optimal solution with respect to $J(R(.))$ in (16), $\left(X^{i^*}(t), R^{i^*}(t)\right), t \in \left[t_0^i, t_1^i\right]$, denote the ith-stage optimal solution with respect to $J^i(R(.))$ in

$$J^i(R(.)) = \int_{t_0^i}^{t_1^i} \left(X^t(t)L(t)X(t) + R^t(t)W^t(Y(t))W(Y(t))R(t) \right) dt + X^t(t_1^i)Q^i X(t_1^i) \tag{17}$$

and $\left(X_\infty^{i^*}(t), R_\infty^{i^*}(t)\right), t \in \left[t_0^i, t_1^i\right]$, be the ith-stage asymptotically optimal solution with respect to

$$J_\infty^i(R(.)) = \int_{t_0^i}^{\infty} \left[X^t(t)LX(t) + R^t(t)W_i^t W_i R(t) \right] dt \tag{18}$$

If N > Ñ, (A_i,B_i) is completely controllable and (A_i,C) is completely observable , for all i = 1,………,r, then

1)
$$\left(X^*(t), R^*(t)\right) = \begin{cases} (X_\infty^{i^*}(t), R_\infty^{i^*}(t)), \forall t \in \left[t_0^i, t_1^i\right], i = 1, \ldots\ldots, N-1 \\ (X^{i^N}(t), R^{i^N}(t)), \forall t \in \left[t_0^N, t_1\right] \end{cases}$$

$$\text{where } t_0^i = t_1^{i-1}, i = 2, \ldots\ldots\ldots, N \;\&\; t_0^1 = t_0; \tag{19}$$

2) For the ith stage, the optimal synthetical control law is

$$R_\infty^{i^*}(t) = -W_i^t \left[W_i W_i^t \right]^{-1} B^t H_i^t \pi_\infty^i X_\infty^{i^*}(t), t \in \left[t_0^i, \infty\right) \tag{20}$$

and the optimal trajectory is

$$\dot{X}_\infty^{i^*}(t) = \left(H_i A - H_i B B^t H_i^t \pi_\infty^i \right) X_\infty^{i^*}(t), t \in \left[t_0^i, \infty\right) \tag{21}$$

where π_∞^i is the unique symmetric positive semidefinite solution of the SSRE

$$A^t H_i^t K + K H_i A - K H_i B B^t H_i^t K + C^t C = 0 \tag{22}$$

3) As for the last stage, the th stage, the optimal synthetical control law is

$$R^{i^*(t)} = -W_i^t \left[W_i^t W_i \right]^{-1} B^t(t) H_i^t \pi^i(t, t_1^i) X^{i^*}(t) \tag{23}$$

and the optimal trajectory is

$$\dot{X}^{i^*}(t) = \left[H_i A(t) - H_i B(t) B^t(t) H_i^t \pi^i(t, t_1^i) \right] X^{i^*}(t), \tag{24}$$

where $\pi^i(t, t_1^i)$ is the symmetric positive semidefinite solution of the segmental Riccati DE

$$-\dot{K}(t) = L(t) - K(t)H_i B(t) B^t(t) H_i^t K(t) + A^t(t) H_i^t K(t) + K(t) H_i A(t) \tag{25}$$

4) The minimum performance index is

$$R_{[t_0,t_1]}^{\min} J\left(R(.)\right) = \sum_{i=1}^{N-1}\left[X_{\infty}^{i^{*t}}(t_0^i)\pi_{\infty}^i X_{\infty}^{i^*}(t_0^i)\right] + X^{N^{*t}}(t_0^N)\pi^N(t_0^N, t_1)X^{N^*}(t_0^N) \tag{26}$$

Infinite-Horizon Problem: Which is the case that the operating time goes to infinity or is much larger than the time-constant of the dynamic system. For the fuzzy system and fuzzy controller in (1) and (2), respectively, if the linearized fuzzy system in (5) is controllable and there exists on $[t_0,\infty)$ an n x n symmetric positive semidefinite solution $\phi(t, t_0)$ to the forward Riccati-like DE

$$\dot{K}(t) = L(t) - K(t)H\left(X(t)\right)B(t)B^t(t)H^t\left(X(t)\right)K(t) - A^t(t)H^t\left(X(t)\right)K(t) - K(t)H\left(X(t)\right)A(t) \tag{27}$$

where $\dot{K} \geq 0$ and the initial value of the dependent variable $K(t_0) = 0$, then there exists a optimal synthetical control law

$$R^*(t) = W^t(Y^*(t)\left[W\left(Y^*(t)\right)W^t\left(Y^*(t)\right)\right]^{-1} \times B^t H^t\left(X^*(t)\right)\phi(t,t_0)X^*(t) \tag{28}$$

which minimizes

$$J\left(R(.)\right) = \int_{t_0}^{t_1}\left[X^t(t)L(t)X(t) + R^t(t)W^t\left(Y(t)\right)W\left(Y(t)\right)R(t)\right].dt + X^t(t_1^i)Q^i X(t_1^i) \tag{29}$$

and the corresponding global minimizer is

$$u^*(t) = B^t H^t\left(X^*(t)\right)\phi(t,t_0)X^*(t) \tag{30}$$

The dynamics of the resultant closed-loop fuzzy system is described by

$$\dot{X}^*(t) = \begin{bmatrix} H\left(X^*(t)\right)A(t) - H\left(X^*(t)\right)B(t)B^t(t) \\ H^t\left(X^*(t)\right)\phi(t,t_0) \end{bmatrix} X^*(t);\quad t \in [t_0, \infty] \tag{31}$$

with $X(t_0) = X_0$.

III. LOCAL CONCEPT VERSUS GLOBAL CONCEPT

The global concept approach first converts the continuous fuzzy system to the linear-like global system representation. The individual matrices are unified into synthetical matrices. Whereas the local concept approach applies "blending" optimal local fuzzy controllers to achieve the global optimal. In the local concept approach, the sufficient condition of the existence of an nxn symmetric positive semidefinite solution Π^i (t, Q, t_1) to the matrix Ricatti differential equation (4) is proposed then the existence of local optimal fuzzy control law $r_i^*(t)$ and corresponding global minimizer $u^*(t)$ is achieved.

In global concept approach the segmental Riccati-like differential equation is solved to find symmetric positive semidefinite solution Π^i (t, t_1) at the i^{th} stage. Then the optimal control law $R^*(t)$ and corresponding global minimizer are determined. The membership function is assumed to be invariant during the whole single stage. The specifically designed algorithm called dynamic decomposition algorithm (DDA) incorporates all the above conditions.

Hence, in global concept approach due to the proposed dynamic decomposition algorithm, the solution to the optimal controller design on computer is achieved conveniently. Whereas in local concept approach the procedure of optimal controller design is a theoretical one.The restrictions on the chosen membership function are applied by the criterion;

$$dH(X^*(\tau)/d\tau \leq k_{H_1} \quad \text{and} \quad \left\|H\left(X^*(t)\right) - H\left(X^*(t_0^i)\right)\right\| < k_{H_2} \tag{32}$$

To ensure that the membership degrees corresponding to optimal trajectory at t_0^i do not change in abrupt shape to check the almost invariant criterion for the entire i^{th} stage. These conditions are considered in dynamic decomposition algorithm, whereas local concept approach does not have the above restrictions.

IV. NUMERICAL SIMULATION

Consider a nonlinear mass-spring-damper mechanical system shown in fig. (1). The system can be formulated as

$$M\ddot{x} + g(x,\dot{x}) + f(x) = \phi(\dot{x})u \tag{33}$$

where M is the mass and u is the force; $f(x)$ and $g(x,\dot{x})$ are the nonlinear or uncertain terms with respect to the spring and the damper, respectively, and **ϕ(x)** is the nonlinear term with respect to the input term[8] . We make the same assumptions as Tanaka *et al.* [8] did and reformulate the system as

$$\ddot{x} = -0.1\dot{x}^3 - 0.02x - 0.67x^3 + u \tag{34}$$

where $x \in [-1.5 \quad 1.5]$ and $x \in [-1.5 \quad 1.5]$.

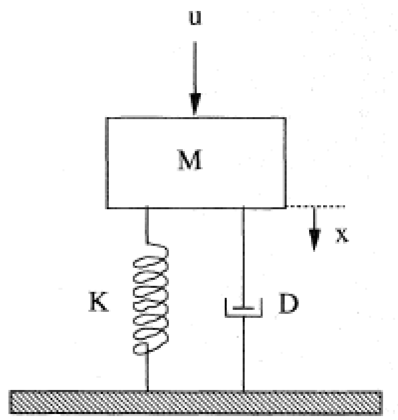


Fig.1 mass-spring-damper mechanical system

Let $X(t) = [x_1(t) \quad x_2(t)]^t = [x_1 \quad x_2]^t$. The system in the above can be described by the following T–S type fuzzy model [17];

$$R^i : \text{IF } x_1(t) \text{ is } F_1^i \text{ and } x_2(t) \text{ is } F_2^i \text{ Then } \dot{X}(t) = A_i(t)X(t) + B_i(t)u(t)$$
$$Y(t) = CX(t), i = 1, \ldots.., 4.$$

where the initial conditions are $X(0) = X_0$ and $Y(0) = CX_0$ with $C = I_2$ for every rule and the membership functions are chosen as

$$\mu_{F1}^1 = \mu_{F1}^2 = 1 - \left(\frac{x_1^2(t)}{2.25}\right), \mu_{F1}^3 = \mu_{F1}^4 = \frac{x_1^2(t)}{2.25}, \mu_{F2}^1 = \mu_{F2}^3 = 1 - \left(\frac{x_2^2(t)}{2.25}\right), \mu_{F2}^2 = \mu_{F2}^4 = 1 - \left(\frac{x_2^2(t)}{2.25}\right)$$

$$A_1 = \begin{bmatrix} 0 & -0.02 \\ 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.225 & -0.02 \\ 1 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & -1.5275 \\ 1 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} -0.225 & -1.5275 \\ 1 & 0 \end{bmatrix}$$

We further assume fuzzy controller given by

$$R^i : \text{IF } x_1(t) \text{ is } F_1^i \text{ and } x_2(t) \text{ is } F_2^i \quad \text{Then} \quad u(t) = r_i(t)$$

Accordingly, the firing-strength of the i[th] rule is $\alpha_i\left(x(t)\right) = \mu_{F2}^2 . \mu_{F2}^4$ and the normalized firing-strength of the i[th] rule is

$$h_i\left(X(t)\right) = \frac{\alpha_i\left(x(t)\right)}{\sum_{i=1}^{4} \alpha_i\left(x(t)\right)} \text{ for } i = 1, \ldots..., 4.$$

The parameters are set as follows; $\mu_{F1}^1 = \mu_{F1}^2 = \mu_{F1}^3 = \mu_{F1}^4 = \mu_{F2}^1 = \mu_{F2}^3 = \mu_{F2}^2 = \mu_{F2}^4 = 0.5$

$$\therefore \alpha_i\left(x(t)\right) = 0.25 \Rightarrow h_i\left(X(t)\right) = 0.25 \quad \text{as} \quad \sum_{i=1}^{4} \alpha_i\left(x(t)\right) = 1, B_i = [1 \quad 0]^t; C = [1 \quad 0], L = I_2$$

With Local Concept approach:

The subsystem has rank$[B_i, A_iB_i]$ = 2 and rank $[C^T, A_iC^T]$ = 2 indicating the subsystem (A_i, B_i) to completely controllable and (A_i, C_i) to completely observable for all i = 1,2,3,4. Substituting the above matrix values to the steady state riccati equation (SSRE) (4) i.e. $A_i^T K + KA_i - KB_iB_i^T K + C^T C = 0$ gives the following valid solution;

For $i = 1; k = \begin{bmatrix} 1.7206 & 0.9802 \\ 0.9802 & 1.7209 \end{bmatrix}$; corresponding local optimal control law from (5) i.e.

$$r_i^*(t) = -B_i^T \pi_\infty^i X^*(t), i = 1,....,4. \text{ is } r_1^*(t) = -\left(0.9802x_1 + 1.7206x_2\right)$$

For $i = 2; k = \begin{bmatrix} 1.5102 & 0.9802 \\ 0.9802 & 1.7311 \end{bmatrix}$; As found earlier, corresponding local optimal control law is

$$r_2^*(t) = -\left(0.9802x_1 + 1.5102x_2\right)$$

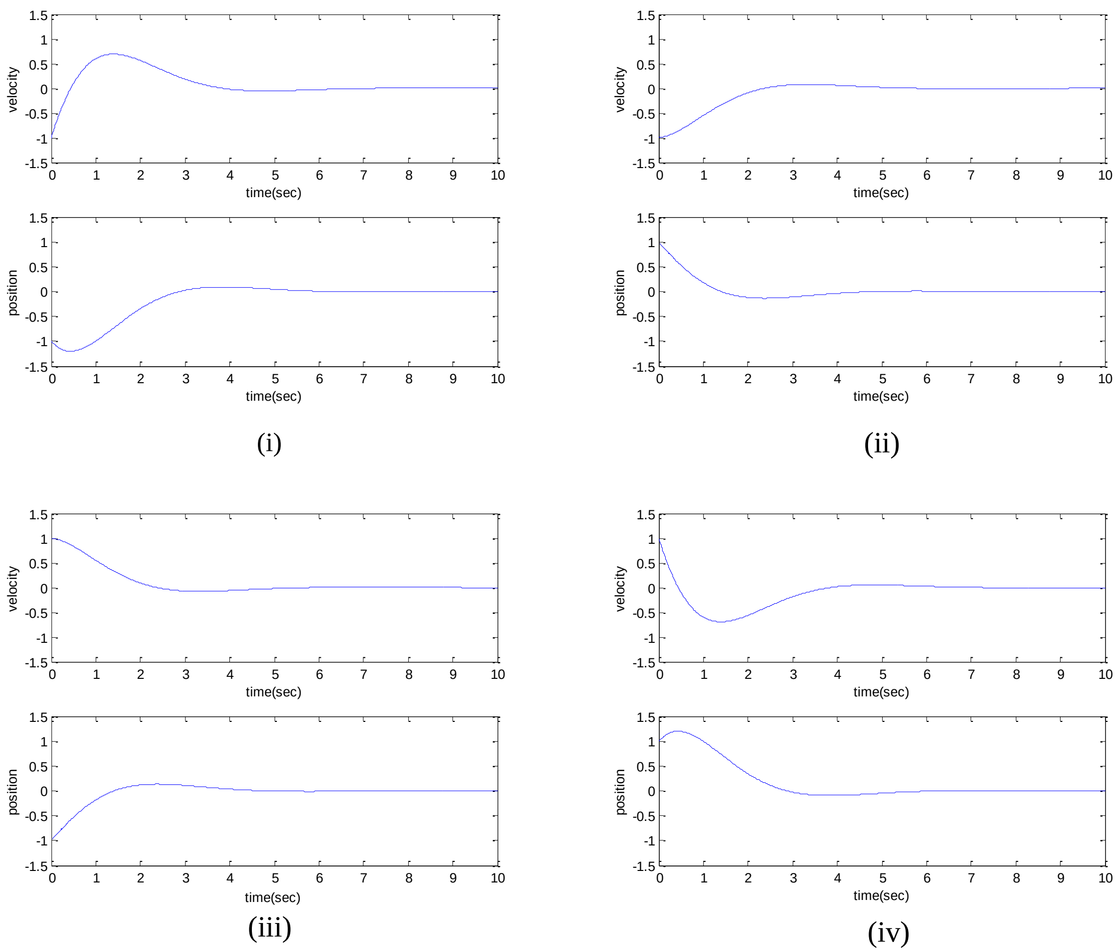


Fig .2 The state responses (position and velocity) of continuous time fuzzy system with designed optimal controller via, local concept approach of optimal fuzzy controller design at the initial condition X(0)= $(-1,-1)^T$, $(-1,1)^T$,$(1,-1)^T$ and $(1,1)^T$ at (i),(ii),(iii) and (iv) respectively.

For $i = 3; k = \begin{bmatrix} 1.2635 & 0.2982 \\ 0.2982 & 2.3068 \end{bmatrix}$; corresponding local optimal control law **is given** as

$$r_3^*(t) = -\left(0.2982x_1 + 1.2635x_2\right)$$

For $i = 4; k = \begin{bmatrix} 1.0584 & 0.2982 \\ 0.2982 & 1.9994 \end{bmatrix}$; corresponding local optimal control law is given as

$$r_4^*(t) = -\left(0.2982x_1 + 1.0584x_2\right)$$

The global optimal control law is given by (6) i.e. $u^*(t) = \sum_{i=1}^{4} A_i(X^*(t)r_i^*(t)$

Substituting $A_1, A_2, A_3, A_4, r_1^*(t), r_2^*(t), r_3^*(t)$ and $r_4^*(t)$ calculated earlier, we get

$$u^*(t) = -\left(0.6392x_1 + 1.3882x_2\right)$$

The optimal closed loop system by (7) is described by $X^*(t) = \left(A_i - B_i B_i^T \pi_\infty^i\right) X^*(t)$ On substituting the A_i, B_i and π_∞^i, we get

$$X^*(t) = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T = \begin{bmatrix} 0 & 1 \\ 1.5007 & -1.4128 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Fig. 2 illustrates the position and velocity responses of the closed-loop fuzzy system in different initial conditions. From the simulation results, we find that the designed optimal fuzzy controller can quickly push the system from various initial states to and stay at the desired states.

With Global Concept approach:
The linear-like dynamical fuzzy system representation for the nonlinear mass-spring damper mechanical system is

$$\dot{X}(t) = H\left(X(t)\right)A(t)X(t) + H\left(X(t)\right)B(t)W\left(Y(t)\right)R(t)$$

$$Y(t) = C(t)X(t) \quad \text{with} \quad A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}; \quad B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}; \quad R = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}$$

$$H\left(X(t)\right) = \left[\, h_1\left(X(t)\right) h_2\left(X(t)\right)\ h_3\left(X(t)\right)\ h_4\left(X(t)\right) \right]$$

$$W\left(Y(t)\right) = \left[w_1\left(Y(t)\right) w_2\left(Y(t)\right) w_3\left(Y(t)\right) w_4\left(Y(t)\right) \right]$$

$$\text{i.e. } A = \begin{bmatrix} 0 & -0.02 \\ 1 & 0 \\ -0.225 & -0.02 \\ 1 & 0 \\ 0 & -1.5275 \\ 1 & 0 \\ -0.225 & -1.5275 \\ 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$H(X(t)) = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix};$$
$$W(Y(t)) = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix};$$

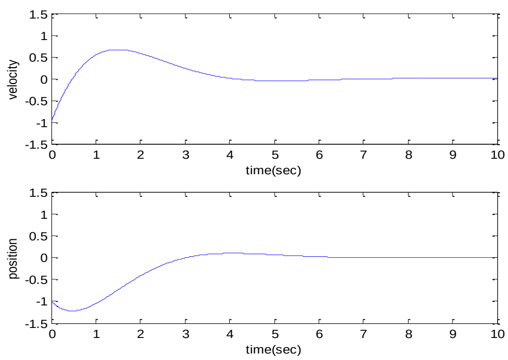
The subsystem has rank[B_i , A_iB_i] = 2 and rank[C^T ,A_iC^T]=2 indicating the subsystem (A_i ,B_i) to completely controllable and (A_i,C_i) to completely observable for all i =1,2,3,4. Substituting the above matrix values to the steady state riccati equation (SSRE) (13), gives the following valid solution;

i.e. $$K = \begin{bmatrix} 1.3 & 0.4905 \\ 0.4905 & 1.7 \end{bmatrix},$$

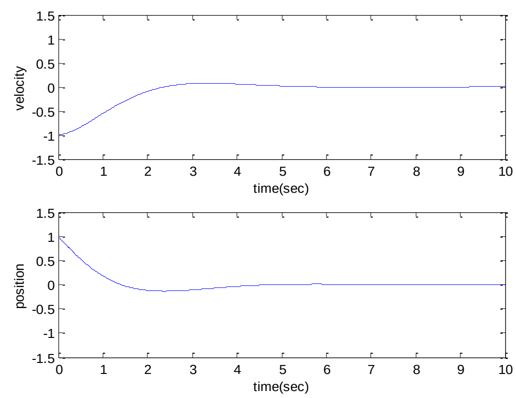
the corresponding optimal control law is given as $u^*(t) = 1.3x_1 + 0.4905x_2$;

and the corresponding optimal state trajectory is given by; $X^*(t) = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T = \begin{bmatrix} 1.412 & -1.2645 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

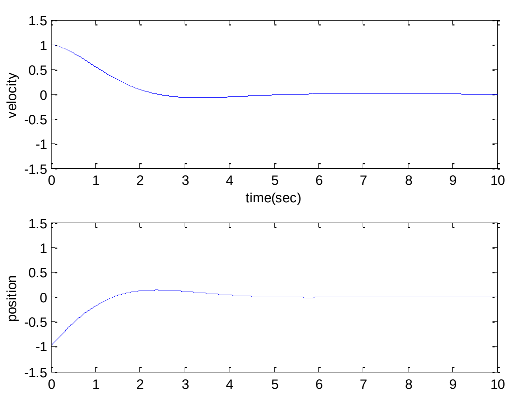
The outputs of the designed optimal fuzzy controller and the state responses of the resultant closed-loop fuzzy system are shown in figure 3, which reveals that the designed optimal fuzzy controller can promptly push the mass-spring-damper mechanical system from various initial states to and stay at the desired states.



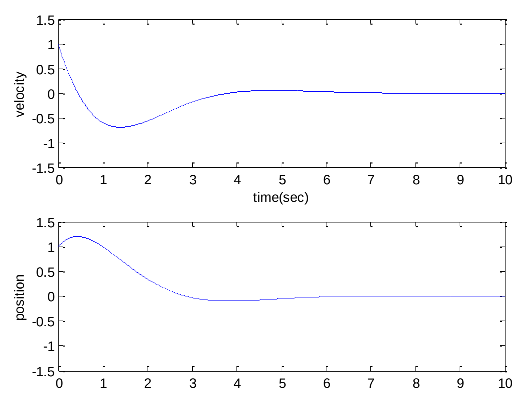
(i)



(ii)



(iii)



(iv)

Fig. 3 The state responses (position and velocity) of continuous time fuzzy system with designed optimal controller via, global concept approach of optimal fuzzy controller design at the initial condition X(0)= $(-1,-1)^T$, $(-1,-1)^T$, $(1,-1)^T$ and $(1,1)^T$ at (i),(ii),(iii) and (iv) respectively.

V. CONCLUDING REMARKS

On comparison of the simulation results of mass-spring-damper mechanical system with the two approaches in figure (2) and figure (3) reveals that the state response with designed optimal controller via, local concept approach and global concept approach are exactly the same. However, the problem of optimal fuzzy controller design has been dealt in a different manner by the two methods. The reviewed global concept approach of optimal controller design is basically the extension of the other reviewed approach i.e. the local concept approach for optimal fuzzy controller design. The global concept approach can be computer adaptive due to the inclusion of dynamic decomposition algorithm and is effective in providing solutions to bigger volume optimal fuzzy controller design problems.

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